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Saul Abarbanel

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During the period covered by the Grants attention has been focused on three areas, all of them of importance in the successful application of implicit algorithms to Computational Fluid Dynamics (CFD):

1. The role of boundary conditions for implicit hyperbolic schemes.
2. The stability of hyperbolic Approximate Factorization schemes in three space dimensions.
3. The rate of convergence to steady state of ADI methods.

This report delineates the progress in each of the above enumerated areas. The details of the research will be found in reports and papers as referenced below for each of the tasks.

1. Role of Boundary Conditions

It is well known that for problems governed by hyperbolic partial differential equations one cannot specify boundary conditions along all the domain. In fact, every point on the boundary which receives information, via the characteristics, from the initial data must be excluded from boundary value specification. On the other hand, in the numerical solution of the approximating difference equations one must often use those "excluded" boundary points in order to advance the integration temporally. (This may be avoided by using upwind differencing; one then, however, may pay a price--e.g. lower spatial accuracy.) The theory of how to treat numerically these "forbidden" hyperbolic boundaries is due to Kriess et al [1,2,3,4,5] and Osher [6,7]. The theory is not easy to apply to systems of equations or in more than one space dimension. In the multidimensional case one must assume periodic solutions in the directions not normal to the boundary being investigated. Application of the G-K-S theory to one-dimensional Beam-Warming type algorithms was carried out by Gustafsson and Oliger [8], and Yee, Beam and Warming [9] for extrapolation-type boundary conditions.

In the present research it was thought interesting to find out whether the conclusions drawn from the one-dimensional analysis carry over to the 2-D case. Some previous work [10] with explicit schemes indicated that this is not always the case. The details of the present investigation were reported at the NASA-Ames workshop on boundary conditions (Oct. 1981), have come out as an ICASE report and were published in the JCP [11]. The major conclusions of this study may be summarized as follows: extrapolation-type boundary conditions which are stable in one dimension will remain so in two dimensions if the indicated extrapolation is carried out in a direction normal to the plane of the boundary. If the extrapolation is done in a non-normal direction (skewed to the boundary plane), then the outcome depends on the nature of the basic internal-points algorithm. If this internal scheme is strictly dissipative (e.g. MacCormack's algorithm), then the 1-D results carry over even if the extrapolation in 2-D be performed in a direction skewed to the boundary plane. If, on the other hand, the internal scheme is not strictly dissipative (e.g. backward Euler), then the 1-D boundary treatment remains stable only if the extrapolation is normal to the boundary plane; otherwise it is unstable (not even conditionally stable).

2. Stability of 3-D Hyperbolic Approximate Factorization Schemes

It has been pointed out by Dwoyer and Thames [12] that the Beam-Warming algorithm in 3-D is unstable for the linear wave equation. It was suspected but not demonstrated that this is also true of the system of linearized equations resulting from Euler's equations of gas-dynamics.

As part of the present research effort it was undertaken first to demonstrate the instability of the Approximate Factorization, backward Euler algorithm when applied to the equations of gas-dynamics (Euler equations). The second task was to try and construct an algorithm which will retain the advantages of the Beam-Warming algorithm (approximate factorization of the implicit terms, "delta form" of the unknown vector), or of the LODQ algorithm (see [12]), and yet become stable due to changes introduced into the explicit terms only. Both tasks were completed successfully and are reported in [13].

The new stabilized algorithm has the following features: three LOD (locally one dimensional) fractional steps are followed by [illegible]plicit step. The total algorithm is stable in 3 dimensions, is spatially second order accurate for a large range of the Courant number when it converges to steady state. The intermediate steps leading to the steady state are not time consistent and hence do not represent a truly evolving solution. This new algorithm when applied to the 2-dimensional case (which is stable for the standard AF methods) accelerates the convergence to steady state by factors between 2 to 3 depending on the mesh size. The numerical experiments carried out in connection with this work brought out the little known (or rather little acknowledged) fact that the rate of convergence of hyperbolic AF algorithms to steady state is very sensitive to the time step. In fact, for the linear wave equation, the optimal, Courant number is of order unity. Similar results for the full Euler equations were also reported by Thompkins and Bush [14]. These empirical observations led us to concentrate on the third topic.

3. Convergence Rate to Steady State of ADI Method

ADI methods for elliptic partial differential equations were proposed already in the 1950's. Particularly well known were the algorithms due to Peaceman and Rachford [15] and Douglass and Gunn [16]. In recent years ADI methods were advocated to solve parabolic and hyperbolic partial differential equations, see Beam and Warming [17], and Briley and McDonald [18]. The motivation for this extension of ADI schemes was to combine the convenience of one-dimensional easily invertible operators and the unconditional stability of implicit methods. As pointed out above, the large expected gains over direct explicit solvers have not been realized fully. This is so because the rate of convergence to steady state is very sensitive to the Courant number and it decreases rapidly, i.e. the iteration count grows rapidly, when the calculation is carried out away from an optional time step. At the optimal Courant number the convergence rate is comparable to those of explicit methods.

The work carried out under the present Grants concentrated so far on the parabolic case which is easier to analyze. The results obtained so far may be summarized as follows (see [19]):

3.1 The convergence to steady state of parabolic ADI solvers, such as the Douglass-Gunn or the Peaceman-Rachford algorithms, is analyzed in terms of the L_2 norm of the residual. This approach, which assumes the presence of many frequencies and averages over their spectrum, turns out to be sudcessful in predicting how the number of iterations needed to converge to steady state depends on the Courant number.

3.2 A new corrected ADI algorithm has been devised which has the following properties:

a) Its construction necessitates only the addition of the same explicit term to all existing Approximate Factorization codes.

b) It is robust in the sense that it need not be fine-tuned for different mesh sizes, different grid stretchings, mixed Dirichlet-Neumann boundary conditions, etc.

c) The rate of convergence to steady state is substantially improved and is insensitive to the Courant number over a large range.

d) Its method of derivation is easily extended to the three-dimensional case.

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